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On the Coexistence of Solar-Energy Conversion and Plant Cultivation

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In this paper we propose a configuration of a solar, e.g., photovoltaic, power plant, which allows for additional agricultural use of the land involved, although the collectors are optimized for solar-energy conversion. If the collectors are not installed directly on the ground, but are elevated by about 2 m above the ground with the periodic distance between collector rows of about three times the height of the collectors, one achieves nearly uniform radiation, (integrated over the day), on the ground of a value of about two-thirds of the global radiation without solar collectors. The mathematical relations allowing calculation of the fraction of light reaching the ground under the collector field are derived. Numerical calculations for both the direct and diffuse part of solar radiation are carried out yielding the seasonal and local dependence of this fraction. In addition, we give an outline of the various advantages offered by this configuration.

1. INTRODUCTION

In the past it has been generally assumed that solar-energy conversion precludes any other use of the land area involved. Thus, large or medium scale solar plants were envisioned only in arid climatic zones. In the more temperate zones of larger geographic latitudes, plant growth, e.g., within a photovoltaic collector area, was considered a nuisance which had to be prevented.[1] Gloomy visions of artificial desert areas due to photovoltaic utilization were advanced. It was even postulated that this type of solar-energy conversion would lead to real estate speculation with concomitant price escalation and dire social consequences.[2] The photovoltaic panels were relegated to the roofs of single family dwellings, which are indeed a prime target for early introduction of this type of energy.[3]

In this paper it will be shown that solar-energy collectors and agricultural use of the same area are very compatible; that this combined use will enhance

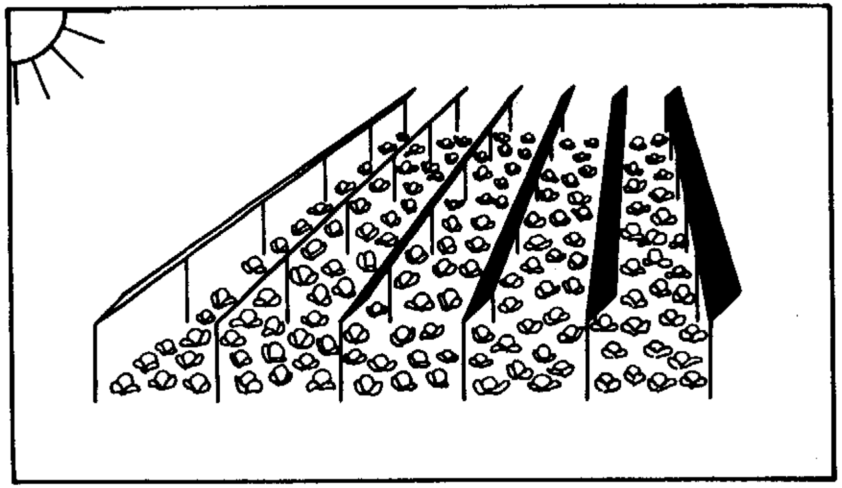

FIGURE 1 Model sketch of elevated collector field.

the benefits extracted from the land. The logic being followed here is based on the following very simple postulates:

- For optimum energy conversion any flat-plate collector has to be directed south (within the northern hemisphere) and *tilted* at an angle equivalent to the geographic latitude or slightly higher.[4]
- To prevent excessive shadowing the collector rows have to be spaced some distance apart, a common rule of thumb being about three times the height of the collector.
- By virtue of this arrangement a large amount of solar radiation will reach the ground between the collector plates. The amount reaching the ground will be particularly large during the summer months, coinciding with the growing season.
- If the collectors are placed directly at ground level, the radiation density between the collectors will be very inhomogeneous, with almost total shadowing underneath the collectors.
- By elevating the collectors on a suitable support structure, the distribution of radiation can be evened out.
- Solar collectors are optimized for good performance in the winter and transition season. They do not intercept the bulk of the light during the other seasons. Thus they match almost ideally with the plant growing season.

We thus arrive at a configuration which is sketched in Figure 1. In the following parts of this paper the theoretical relations for the insolation underneath the collector will be developed. A different approach has to be used for direct and diffuse radiation. A numerical evaluation will be carried out for a few sample parameters and applied to actual weather data. The results indicate that, for a latitude of 48°, about two-thirds of the global radiation remains for plant cultivation. Quite a number of crop plants could be grown under these conditions.

The type of combined utilization described here is particularly attractive for photovoltaic conversion. According to present forecasts and actual trends, the manufacturing cost of photovoltaic systems will reach levels permitting competitive energy costs by 1986 or slightly thereafter.[5]

2. THEORY

The mathematical relations derived here are based on the following assumptions: (i) The periodic collector field is facing due south and is of infinite extension in two dimensions; (ii) the stilts on which the collectors are mounted do not contribute any shadowing; (iii) the collectors, including their rear sides, are ideal light-absorbers, (this is not necessarily so, for further details see Section 4); and (iv) the diffuse radiation is isotropic with respect to the hemisphere of the sky. (This is not quite correct because it is known that diffuse radiation is peaked in the direction of the sun; this part would, however, behave like the direct radiation which is also treated here.)

For direct and diffuse radiation the calculation will be quite different. In each case, however, we will evaluate the ratio of radiation arriving at the plane of the ground, with and without collectors.

2.1 Direct sunlight

In the case of direct sunlight, the shadow of each individual collector will move across the ground according to the elevation of the sun. We now calculate the fractional insolation on the ground, integrated over an entire day, which is uniform along the x and y coordinates, (i.e., provided the collectors are raised high enough above ground). The numerical value of integrated fractional insolation, for each strip between two collectors, on the other hand, is independent of elevation. Thus, we may carry out this part of the calculation as if the collectors were mounted directly on the ground, as presented in Figure 2.

Figure 2 contains the definitions of symbols used throughout this paper: (i) a is the dimension of the collectors, this is the only dimension they have since they are assumed to be infinite in the other direction; (ii) d is the distance between two rows of collectors, or the period of the structure; (iii) Θ' is the zenith angle of the sun projected onto plane of the meridian, Θ is the real zenith

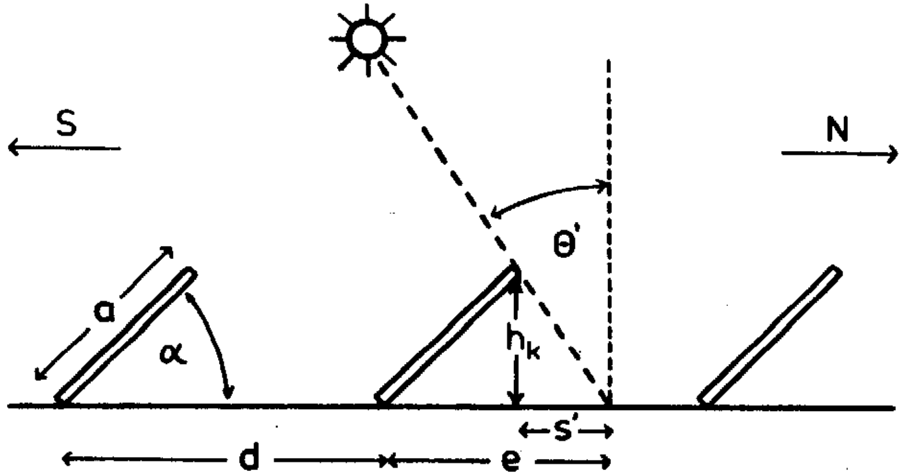

FIGURE 2 Definition of parameters used for calculation of integrated direct radiation.

angle; (iv) e is the length of the shadow of each collector in the same projection; and (v) h_k is the projection of the collector onto a vertical plane.

The quantity of interest is the fractional insolation which is $(d-|e|)/d$. The absolute value of the quantity e has to be taken because if the sun is to the north, the shadow may be thrown in a southerly direction, (Figure 2). e is given by

$$e = a\cos\alpha + s' \tag{1}$$

The projection of the shadow, s', is obtained as shown in Figures 3(a) and 3(b). Figure 3(a) shows the shadow s which is thrown by a body of height h_k, in the plane of the radiation. s' is then the projection of s into the north–south direction (Figure 3b). ω is the hour angle of the sun which is assumed to be the same if projected into the horizon. (This approach breaks down in the tropics where the sun may be directly overhead.)

Thus from Figure 3(a):

$$s = h_k \tan\Theta = a\sin\alpha\tan\Theta, \tag{2}$$

From Figure 3(b) and Eq. (2):

$$s' = s\cos\omega = a\cos\omega\sin\alpha\tan\Theta. \tag{3}$$

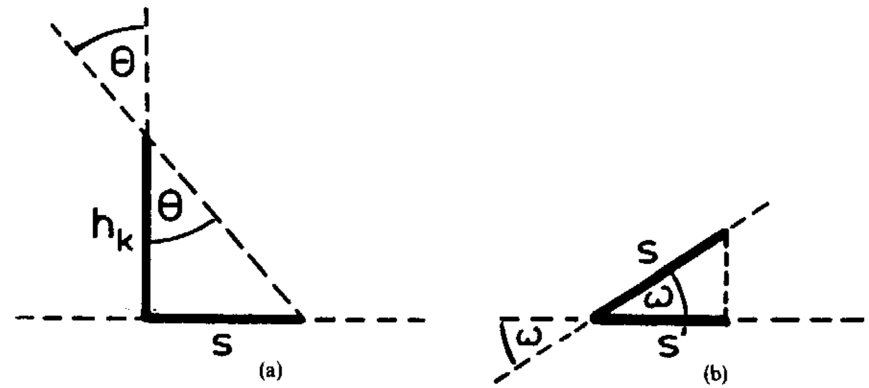

FIGURE 3(a) The vertical projection h_k of the collector causes shadow of length s at zenith angle Θ of the sun. (b) s' is projection of s- into x-direction.

The area exposed to the sun is thus:

$$d-|e| = d - |(a\cos\alpha + s')| =$$
$$= d - |(a\cos\alpha + a\cos\omega\sin\alpha\tan\Theta)| \tag{4}$$

In order to obtain the energy received by the unit area, Eq. (4) has to be multiplied by $\cos\Theta$ (Lambert's law). The ratio of insolation, with and without collectors, is now obtained by integrating from sunrise to sunset:

$$R_{\text{dir}} = \frac{\int_{\omega_R}^{\omega_s} (d-|e|)\cos\Theta\, d\omega}{d\int_{\omega_R}^{\omega_s}\cos\Theta\, d\omega} \tag{5}$$

where, ω_R is the hour angle for sunrise and ω_s that for sunset. Since Eq. (5) is only valid for $d-|e| \geq 0$, the integration limits have to be adjusted to correspond to $d-|e|=0$ whenever necessary.

For $\cos\Theta$ the following relation holds:[6]

$$\cos\Theta = \sin\varphi\sin\delta + \cos\varphi\cos\delta\cos\omega \tag{6}$$

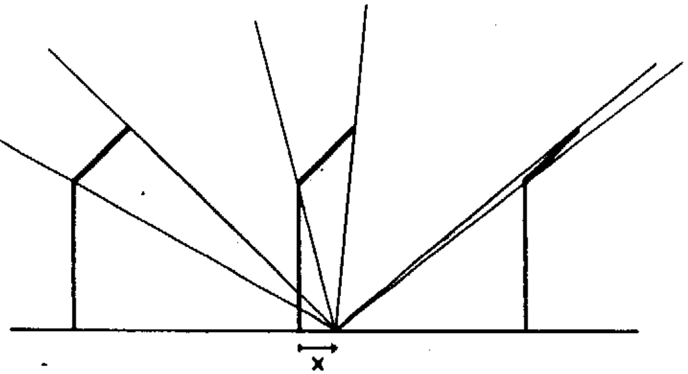

FIGURE 4 Shadowing of diffuse radiation at point x.

with φ = geographic latitude, δ = declination of the sun. For δ we have further:

$$\delta = -23.5\cos\left(\frac{2\pi}{365}(n+10)\right) \tag{7}$$

where n is the number of the calendar day.

The numerator of Eq. (5) has to be integrated numerically, while the denominator can be integrated in closed form. Thus we end up with:

$$R_{\mathrm{dir}} = \frac{\int_{\omega_R}^{\omega_s} (d\cos\Theta - |(a\cos\alpha\cos\Theta + a\sin\alpha\cos\omega\sqrt{1-\cos^2\Theta})|\,\mathrm{d}\omega}{d\sin\varphi\sin\delta(\omega_s-\omega_R) + d\cos\varphi\cos\delta(\sin\omega_s - \sin\omega_R)} \tag{8}$$

2.2 Diffuse light

As stated above, the distribution of the diffuse light is assumed to be uniform over the hemisphere of the sky. Therefore, there is no variation with time of the day relative to an unshaded field. On the other hand, the distribution of diffuse light on the ground will vary along the north–south direction. This distribution will now be evaluated. For this purpose, we calculate how much unobstructed sky can be seen from a given point along the north–south axis, (the x-direction in Figure 4).

The shaded areas, as well as those in between, are lenticular in shape.[7] A cross-section through the north–south plane is shown in Figure 5. The two angles ω_1 and ω_2 are:

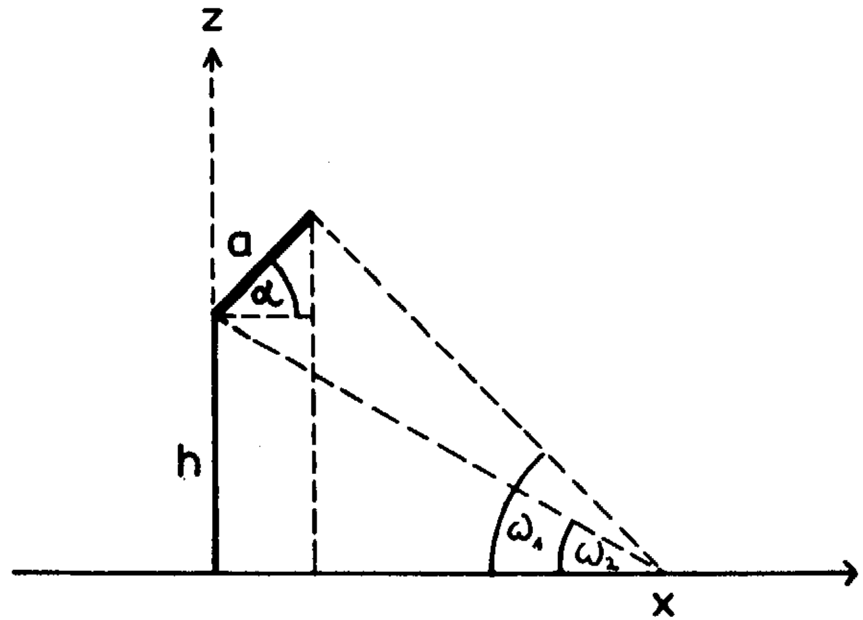

FIGURE 5 Cross section through collector structure with definitions of symbols used for calculation of diffuse radiation.

$$\tan\omega_1 = \frac{h + a\sin\alpha}{x - a\cos\alpha}$$

$$\tan\omega_2 = \frac{h}{x}. \tag{9}$$

The shaded parts of the sky will now be obtained by integration in three-dimensional polar coordinates, which are defined in Figure 6(a). Angle φ is in the plane of the horizon and ω is orthogonal to it. The conditions in the x–y plane are given in Figure 6(b), and those in the plane containing angle ω in Figure 6(c). From Figure 6 we deduce the following relations:

$$y = x\tan\varphi;$$

$$\tan\omega = \frac{z}{\sqrt{x^2+y^2}} = \frac{z}{x\sqrt{1+\tan^2\varphi}} = \frac{z\cos\varphi}{x}. \tag{10}$$

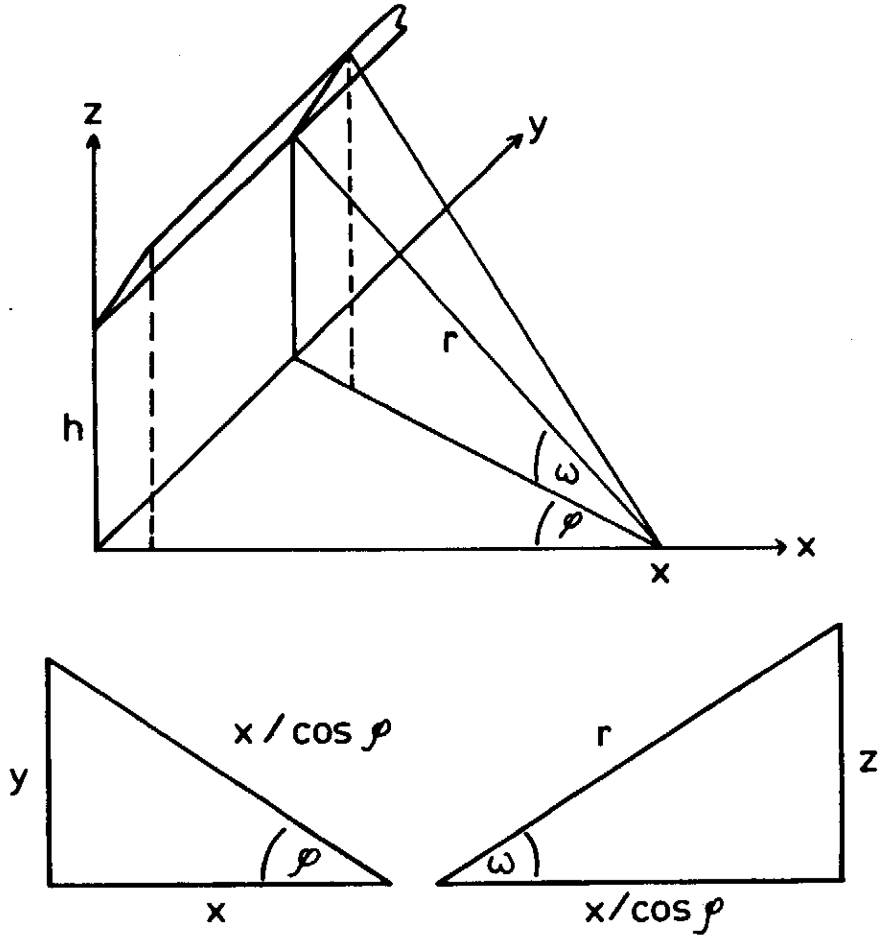

FIGURE 6 (a) Definition of three-dimensional polar coordinates. (b) x–y-plane. (c) Plane containing r and ω.

The limits of integration are the following:

$$\tan\omega_1 = \frac{(h + a\sin\alpha)\cos\varphi}{(x - a\cos\alpha)}$$

$$\tan\omega_2 = \frac{h\cos\varphi}{x}. \tag{11}$$

Now we can express the relative shading by one row of collectors:

$$R_{\mathrm{dif}} = \frac{\int_{-\pi/2}^{+\pi/2}\int_{\omega_2}^{\omega_1} \sin\omega\cos\omega\, d\omega\, d\varphi}{\int_{-\pi/2}^{+\pi/2}\int_{0}^{\pi/2} \sin\omega\cos\omega\, d\omega\, d\varphi}. \tag{12}$$

The term $\sin\omega\cos\omega$ under the integral, results from the differential $\cos\omega d\omega$, of the polar coordinates, times another $\sin\omega$, due to Lambert's law. The denominator can be readily integrated and yields $\pi/2$. The numerator can be integrated with respect to ω:

$$R_{\mathrm{dif}} = \frac{1}{\pi}\int_{-\pi/2}^{+\pi/2} (\sin^2\omega_1(\varphi) - \sin^2\omega_2(\varphi))d\varphi. \tag{13}$$

The limits of Eq. (11) are valid for shading by a collector to the left of x (Figure 4). For collectors to the right of x, we arrive by similar reasoning at the limits:

$$\tan\omega_1 = \frac{(h + a\sin\alpha)\cos\varphi}{d - x + a\cos\alpha}$$

$$\tan\omega_2 = \frac{h\cos\varphi}{d - x}. \tag{14}$$

Absolute values of the integrals have to be taken because if x is close to d, ω_1 is less than ω_2.

In order to arrive at the total shading by a group of collectors, Eq. (13) has to be evaluated numerically for each collector. This is carried out for the more distant individual collectors only until the shadows of succeeding collectors start to overlap. Thus, the number of integrations is rather limited for each x.

3. RESULTS

3.1 Direct light

The numerical calculations in this paper were carried out for a geographic

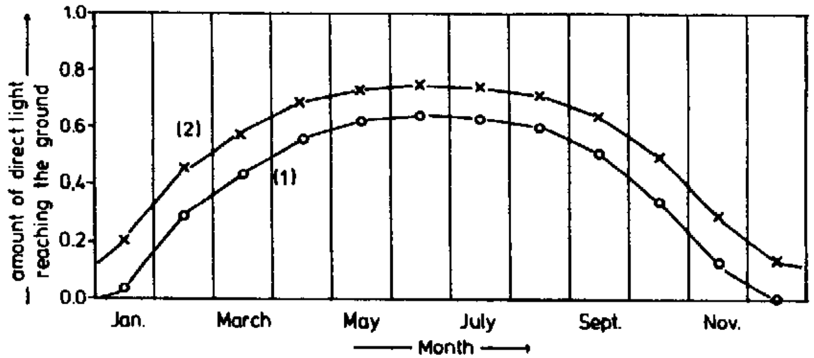

FIGURE 7 Relative insolation on the ground, for the two conditions given in the text, versus months. Unobstructed plane is always at unity.

latitude of 48° and two collector configurations:

1) $a=1$; $d=3$; $h=2$; $\alpha=48°$
2) $a=1$; $d=4$; $h=2$; $\alpha=58°$

The dimensions of a, d and h can be interpreted as meters, but they are scaleable in an arbitrary manner without influencing the results.

The function R_{dir}, from Eq. (8), is plotted in Figure 7 for the two starting conditions. We recognize that for condition 1, the direct light varies from 64% in summer, to 0 in winter; for condition 2, from 75% in summer to 11% in winter, compared to the case without collectors.

3.2 Diffuse light

For the same conditions, labelled 1 and 2 above, Eq. (13) was evaluated. The amount of diffuse light intercepted by the collectors does not depend on seasons, but there is a dependence on the north–south coordinate x. For an infinite array of collectors this function is, of course, periodic with period d.

In Figure 8 the results of the computation are shown. It can be seen that, for the two configurations chosen here, the variation with distance is quite small; in case 1 it is (69 ± 2)%, and in case 2 it is (77 ± 4)%. Since the variations are not very significant we conclude that the distribution of light is approximately uniform. In the following curves, involving actual weather data, we will therefore use the mean values of the diffuse light for cases 1 and 2.

3.3 Application to actual weather data

The above results are now applied to data obtained from the meteorological office in Freiburg, which is at 48° latitude. Figure 9 shows the data used,

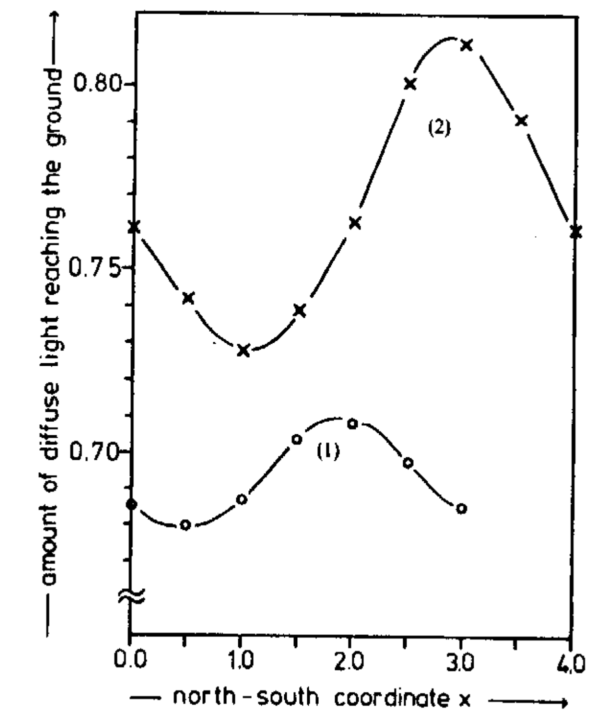

FIGURE 8 Dependence of the amount of diffuse radiation on distance between two collectors.

consisting of global radiation and its direct and diffuse constituents. The data represent a two-year-average from the years 1978 and 1979†. For the direct and diffuse components, the mathematical tools developed in section 2 are now applied and the results presented in Figure 10. This figure shows the total global radiation per unit area, and the amount reaching the ground for the two configurations chosen in this paper. It can be recognized that the global irradiation under the collector field is 67% (case 1) and 76% (case 2), in summer, and 47% (case 1) and 56% (case 2), in winter, compared to an unobstructed level plane. Integrated over the entire year this radiation is 62% (case 1) and 71% (case 2).

It is worth pointing out that a calculation like this is also a way of obtaining the radiation received by a collector field of arbitrary tilt and spacing on a seasonal basis. Figure 11 presents these data which are obtained from Figure 10. It should be kept in mind that these curves are somewhat different from

† It is noticeable that the data of Figure 9 exhibit very low irradiation in the winter months, the ten-year average being much higher. Such a two-year low is, however, within the normal fluctuation.

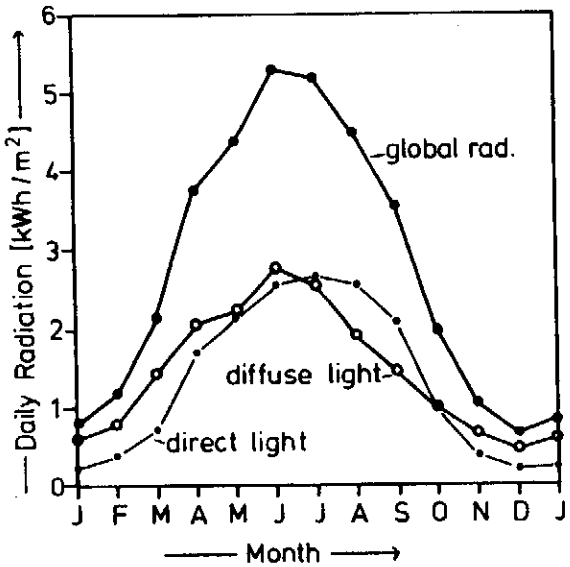

FIGURE 9 Weather data used for numerical computation. (Courtesy Wetteramt Freiburg).

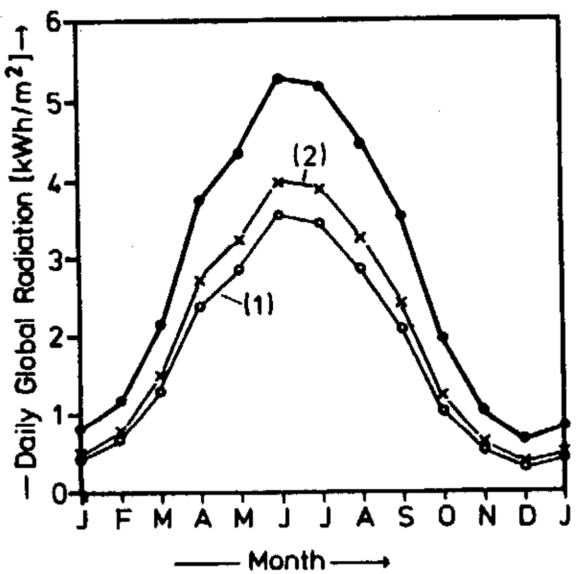

FIGURE 10 Results obtained by applying the mathematical models derived in this paper. Curves shown are, global radiation within collector field for both case 1 and case 2, and total global radiation.

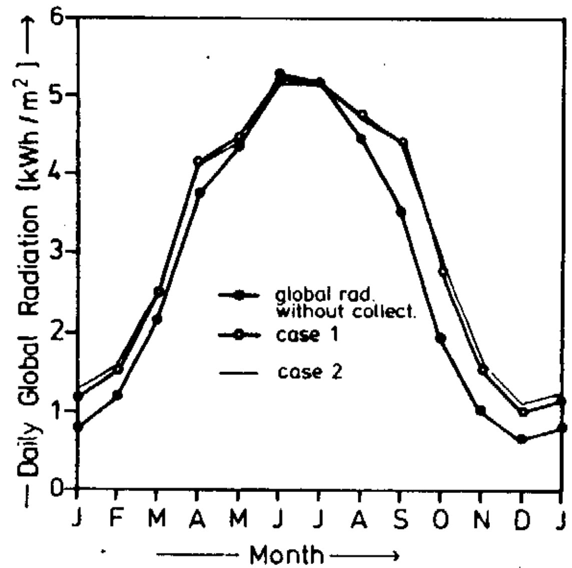

FIGURE 11 Radiation intercepted by collectors. Total global radiation is given per m^2 of surface area and radiation on collectors in terms of collector area.

those for a single-unit collector, since there is some shading of adjacent units, particularly during the winter season. Another factor not contained in Figure 11, is the small amount of light striking the rear sides of the collectors. The approximate yield of solar energy, excluding collector efficiency, is 1188 kWh/m^2 year for case 1, and 1199 kWh/m^2 year for case 2, per m^2 of collector area.

4. DISCUSSION

The major conclusion of this paper is expressed in Figure 10. About two-thirds of the radiation is still available for other purposes, even if the arrangement of the collectors is optimized for solar collection. This is particularly true for condition 1. In this case, the area of the collectors is one-third of the area of land used, and the radiation collected is about one-third of the total. The major advantage of this type of arrangement is, however, a much more even distribution between summer and winter. If weather data with a higher global radiation in winter had been chosen, this would have been more pronounced. Condition 2 is even more optimized for collection during the winter season, which can also be deduced from Figure 10. Of course it is possible to choose an even wider spacing of collectors if the trade-off between solar and agricultural harvesting points towards a different optimization.

Another way of looking at the problem treated here is suggested. If solar energy collectors were to be established in temperature climates undesired plants growing between collectors would have to be kept under control. Why not use the area to cultivate useful crops? The distances between collector rows could be made wide enough to admit mechanized agricultural equipment.

The exchange of radiation between collectors and plants or soil has not been dealt with because it depends too much upon the special circumstances. It can be stated that the collectors, if properly designed, would absorb the light very well but if light were to strike at a glancing angle there would be considerable reflection towards the ground. *Vice versa*, light might be reflected from plants or soil towards the collector. Additional light on the ground and on other collectors would be gained if the rear sides of the collectors were made reflective. Mirrors may be impractical, but simply painting the collectors white would have the same effect.

We now address the question, which type of solar-energy conversion should be considered in the context described here? As stated in the introduction we think the most attractive one is photovoltaics, i.e., solar cells. Studies have shown that an attractive, first application of photovoltaic energy will be on the roofs of single-family dwellings with grid interconnection.[3] This application gives a general idea of the smallest size unit which might be interconnected with the grid in an economical framework. Thus, photovoltaic-agricultural farms of the type investigated here need not be of enormous size, but could be established on an area the size of an average agricultural field. This type of solar farm would in general not be connected directly with an electrical load, but would produce directly for the grid. Thus, it would probably be owned and maintained by the utility. It is also worth mentioning that if it is assumed that both photovoltaic panels and an agricultural crop (involving plants which can be grown with reduced insolation) are economical by themselves on a given piece of land, then the only additional cost involved in the combination of the two is that of elevating the panels on a sufficiently stable support structure.

The final topic to be considered here is, which crops could or should be grown between photovoltaic collectors. This question will be treated in a very cursory manner here because it is a matter of biology or agriculture. A preliminary discussion resulted already in a number of suggestions.[8] As a first principle one should not consider plants whose growth is limited by the availability of light. They are mostly those with C_4 metabolism. Plants which could coexist with collectors are rye, barley, oats or particularly sugar beets. Another attractive proposition which might be considered in certain areas, is that of grazing by livestock such as sheep, game or even cattle, if the collector structure is made sturdy enough.

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